

SEMI-EMPIRICAL MODEL FOR HELICAL MAGNETOCUMULATIVE GENERATORS

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Abstract

A semi-empirical model of helical magnetocumulative generators (MCGs) has been developed and will be presented in this paper. A single adjustable variable, called the characteristic time, has been identified and its physical significance of this variable will be discussed. The model has been applied to four different helical MCGs and the calculated results are in good agreement with the experimental results (See Paper P1-E29 of this conference).

I. INTRODUCTION

The model presented in this paper describes the time dependent changes in the output current of helical generators. This semi-empirical model appears to be general enough to serve as an initial design code. Its simplicity and speed make it practical for doing parametric studies of various types of helical generators with different types of loads.

II. HELICAL MCG MODEL

All the above-mentioned generators can be represented by a series LR-circuit described by the differential equation:

$$\frac{d}{dt}[(L_G + L_L)I] + RI = 0, \quad (1)$$

where L_G and L_L are the inductance of the generator and load, respectively, R the resistance, I the current, and t is time. The inductance of the generator and the resistance will vary in time, while the inductance of the load remains constant. Integrating this equation yields:

$$I = I_0 e^{-g(t)}, \quad (3)$$

where

$$g(t) = \int_0^t \left[\frac{\frac{d(\ln L_G)}{dt} + \frac{R}{L_G}}{1 + \frac{L_L}{L_G}} \right] dt \quad (4)$$

and I_0 is the initial current.

The inductance of the coil in the MCG is determined by using:

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 \pi N^2}{l} (r_c^2 - r_a^2), \quad (5)$$

where μ_0 is the permeability of free space, N is the number of turns, l is the length of the coil, and r_c and r_a are the radii of the coil and the armature, respectively, and the number of turns in the coil can be written in terms of its length and pitch, $N = l/p$. As the armature expands and successively short-circuits the turns in the coil, the length of the coil is shortened. Assuming that this can be described by the equation:

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$$\frac{dl}{dt} = -\frac{l}{\tau}, \quad (7)$$

where τ is a characteristic time, and integrating Eq. 7 yields:

$$l = l_0 e^{-\frac{t}{\tau}}. \quad (8)$$

The inductance is defined by:

$$L = \frac{\mu_0 \pi l_0}{p^2} (r_c^2 - r_a^2) e^{-\frac{t}{\tau}} = L_0 e^{-\frac{t}{\tau}}. \quad (9)$$

The resistance can be written in the form:

$$R = \frac{l}{\sigma A_R}, \quad (10)$$

where σ is the electrical conductivity, A_R is the area of the region contributing to the resistance, and l is the length of the coil. Substituting Eq. 8 into this equation yields:

$$R = \frac{l_0}{\sigma A_R} e^{-\frac{t}{\tau}} = R_0 e^{-\frac{t}{\tau}}. \quad (11)$$

Substituting Eqs. 9 and 11 into Eqs. 3 and 4 yields:

$$I = I_0 \exp \left\{ \left(1 - \frac{R_0 \tau}{L_0} \right) \left[\frac{t}{\tau} + \ln \frac{1 + \frac{L_L}{L_0}}{1 + \frac{L_L}{L_G}} \right] \right\}. \quad (12)$$

It is assumed that,

$$\frac{R_0 \tau}{L_0} \ll 1, \quad (13)$$

for the devices under consideration. This condition indicates that initially the current in the MCG is increasing as predicted by Eq. 12 and verified experimentally.

From a physical point of view, it is clear that at some time t_0 , the current must stop increasing and start to decrease. The time at which this occurs should be related to the time at which the detonation ends. This means that Eq. 12 must be modified. Empirically, it can be shown that the experimental data is represented, to a reasonable degree of accuracy, by the equation:

$$I = I_0 \exp \left\{ \left(1 - \frac{R_0 \tau}{L_0} e^{b(t-t_0)} \right) \left[\frac{t}{\tau} + \ln \left(\frac{1 + \frac{L_L}{L_0}}{1 + \frac{L_L}{L_G}} \right) \right] \right\}, \quad (14)$$

where $t > t_0$ and $b \equiv 1.8 \times 10^5 \text{ sec}^{-1}$. It should be noted that when $t = t_0$, Eq. 14 reduces to Eq. 12.

For the purposes of this study, it has been determined empirically that the initial resistance for each of the generators is given by the relation:

$$R_0 = \frac{91 l_a}{I_0}, \quad (15)$$

where l_a is the length of the armature. The constant in Eq. 15 has the units of electric field (V/m) and $91 l_a$ is an equivalent voltage.

The time t_0 is given by the relation:

$$t_0 = \frac{l_a}{v_D}, \quad (16)$$

where v_D is the detonation velocity. In other words, t_0 is the time at which the detonation terminates.

The initial inductance is given by a series of equations of the same form as in Eq. 9, that is:

$$L_0 = \mu_0 \pi (r_c^2 - r_a^2) \sum_{k=1}^K \frac{l_k}{p_k^2}, \quad (17)$$

where K is the number of induction coils.

III. Characteristic Time

In the above analysis, the only parameter adjusted empirically in order to fit the calculated data to the experimental data is the characteristic time, τ . The values of the characteristic time for the helical generators considered are: Mark IX – 48.0 μs , EF-3 – 31.0 μs , Flexy I – 29.2 μs , and Ranchito – 31 μs . The calculated results are very sensitive to the value of τ . One of the objectives of this paper is to begin to understand what physical factors influence this parameter. Some of these factors include geometrical sizes and shapes, material properties, and detonation characteristics.

As a first step in determining what factors influence τ , it is written in the following form:

$$\tau = \frac{l_c}{v_c}, \quad (18)$$

where l_c and v_c are a characteristic length and velocity, respectively. As the detonation wave proceeds down the armature, the armature expands with some velocity until it hits the coil. Thus, the vertical distance that the armature travels is $l_c = r_c - r_a$. The expanded armature forms an angle α with the axis of the generator. Therefore, the characteristic velocity is:

$$v_c = f v_D \tan \alpha, \quad (19)$$

where the fraction f has been introduced to take into account that only a fraction of the detonation energy is transferred into motion of the armature. Therefore, the characteristic time is:

$$\tau = \frac{r_c - r_a}{f v_D \tan \alpha}. \quad (20)$$

The value of f depends on the mass of the liner and the mass of the explosives available to provide the energy for bending and moving the armature. As a first approximation, the following relation will be used:

$$f = \left(\frac{\frac{C}{M}}{1 + 0.5 \frac{C}{M}} \right)^{\frac{1}{2}}, \quad (21)$$

where C and M are the mass of the explosive and the armature, respectively. This relationship was developed by Gurney [1] in his analysis of the velocity of fragments from an explosion.

In the Mark IX experiments, 60 kg of PBX 9501 was used and the armature was made of copper. Using the dimensions of the armature given in [2], the calculated mass of the armature is 71.9 kg, which means that $C/M = 0.834$ and $f = 0.767$. Using these values and a characteristic time of 48 μsec , the angle α is calculated to be 15.7° . The measured value given in [2] is 14° .

For the Flexy I, the armature was made from aluminum. Using the dimensions given in [3], its mass is calculated to be 8.29 kg. The mass of the explosives was given as 15 kg. Therefore, $C/M = 1.81$ and $f = 0.975$. For a characteristic time of 29.2 μsec , the angle α is 12.8° . The measured value given in [3] is 12° .

For the EF-3, the mass of the explosives used was not given, but the angle α was given in [3] as 13° . The armature was made of copper and its mass is calculated to be 9.75 kg. Assuming that the mass of the explosives is 7 kg, it is found that $C/M = 0.72$ and $f = 0.727$. Using these values and $\alpha = 13^\circ$, the characteristic time is calculated to be 31 μsec .

IV. SUMMARY

A semi-empirical model of helical MCGs has been developed and benchmarked by using it to calculate the output parameters of generators having a wide range of design and operating conditions. In deriving the model, a sensitive adjustable parameter, called the characteristic time, was identified. This parameter is influenced by factors such as geometrical sizes and shapes, material properties, and detonation characteristics. Relations were developed that show how these factors affect the characteristic time and, thus, operation of the generators.

V. REFERENCES

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